Weather Observation and Analysis

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Course Notes

Chapter 10. GEOSTROPHIC BALANCE

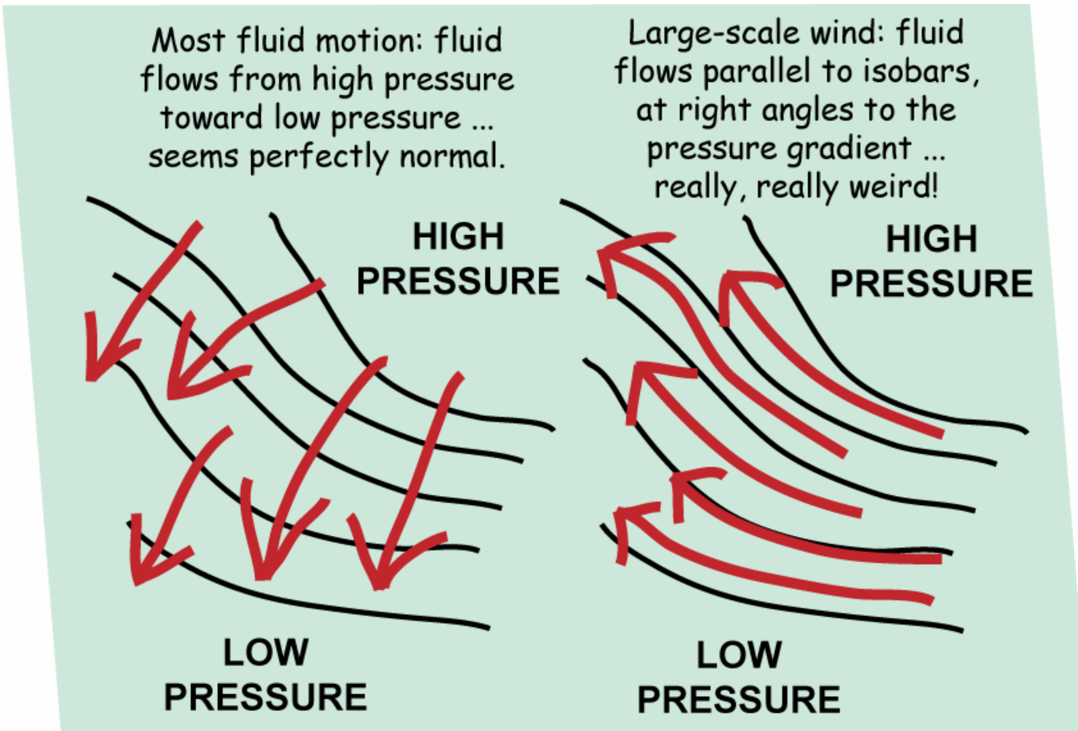
10.1 Wind and Height Gradients

You hear it frequently in weather reports, you've seen it on weather maps, and it makes intuitive sense: strong pressure gradients are associated with strong winds. That statement is a general one that applies to all classes of fluid motion. But if you have a friend who's an engineer or physicist and you want to freak them out, tell them about the atmosphere. Sure, strong gradients mean strong winds, but generally the wind doesn't blow from high pressure toward low pressure. Instead, it blows sideways.

This characteristic is observed in its purest form aloft, away from the effects of surface friction. Most of the time, meteorologists use pressure coordinates when considering the distribution of weather elements aloft, and on a pressure surface, gradients of geopotential height have the same meaning and effect as gradients of pressure at a constant altitude. So from here on, we'll be talking about strong height gradients and the wind not blowing from high heights toward lower heights (on a constant pressure surface such as 500 mb).

On a map of a pressure surface, the height information is usually depicted with contours, just like isobars depict pressure information on a sea level map. If you examine any pressure surface, from 850 mb on up, you will find the following general characteristics:

1. Stronger winds tend to occur where the height contours are closer together; weaker winds where the height contours are farther apart.



2. The winds tend to mostly blow parallel to the height contours.

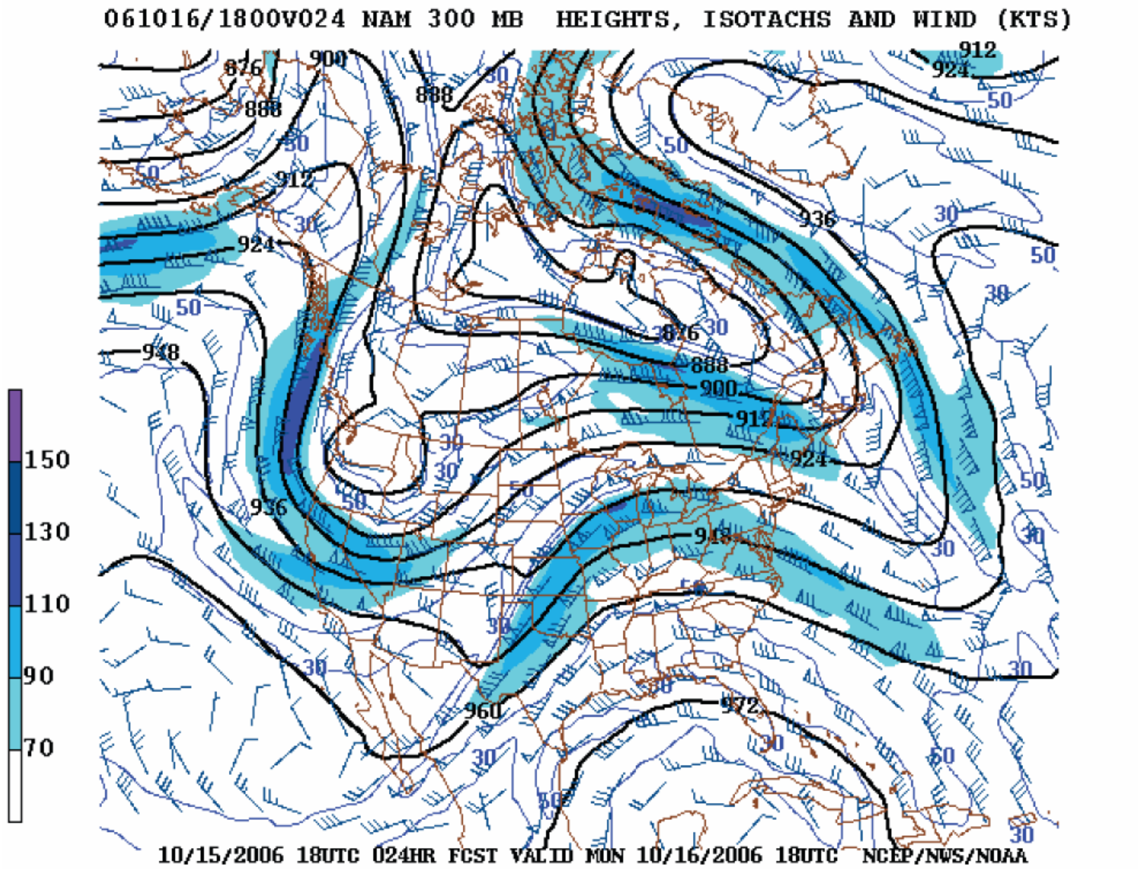
3. The winds tend to be oriented so that the higher heights are 90 degrees to the right of the direction toward which the wind is blowing.

These rules aren't followed exactly, but they are close enough that if you just had the height contours to go by, you could make a pretty good estimate of the wind speed and direction just about everywhere.

A hypothetical wind that follows these rules exactly is called a *geostrophic wind*. Since the real wind comes close in most cases, it's fair to say that the actual wind is nearly geostrophic.

10.2 The Geostrophic Equation

If you were, in fact, confronted with a geopotential height analysis and asked to estimate the wind, you could obtain an exact estimate of the direction pretty easily, by following rules 2 and 3 above. The challenge with estimating the speed is that you need some mathematical formula that tells you exactly how strong a height gradient corresponds to how strong a geostrophic wind speed.



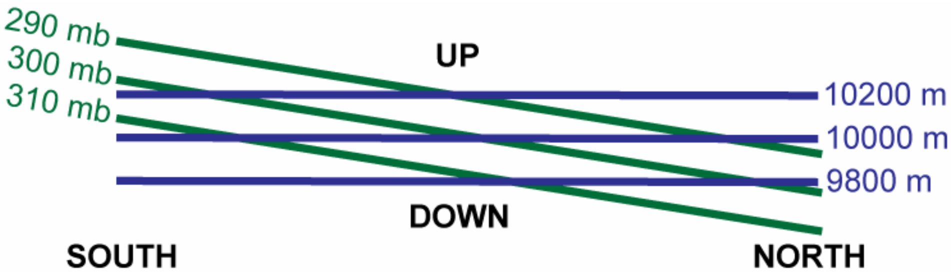
Winds, wind speeds, and height contours on the 300mb surface. The heights are the black contours; heights are labeled in dekameters (1 dam = 10 m). Wind speeds are contoured and shaded in blue, in knots. Notice how the wind barbs are nearly parallel to the height contours throughout the map, and how the higher wind speeds (shown by shading) tend to be where the strongest height gradients are present, that is, where the height contours are closest together. The higher heights are always to the right, facing downstream.

This formula is

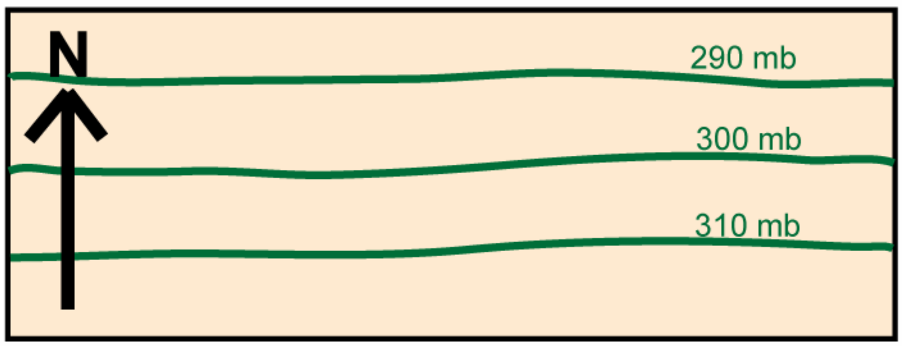
$$\left|\vec{v}_g\right| = \frac{g_0}{f}\left|\nabla_h Z\right|$$

In words, the magnitude of the geostrophic wind is proportional to the magnitude of the geopotential height gradient on a constant pressure surface, with the constant of proportionality being the ratio of the gravitational acceleration and the Coriolis parameter.

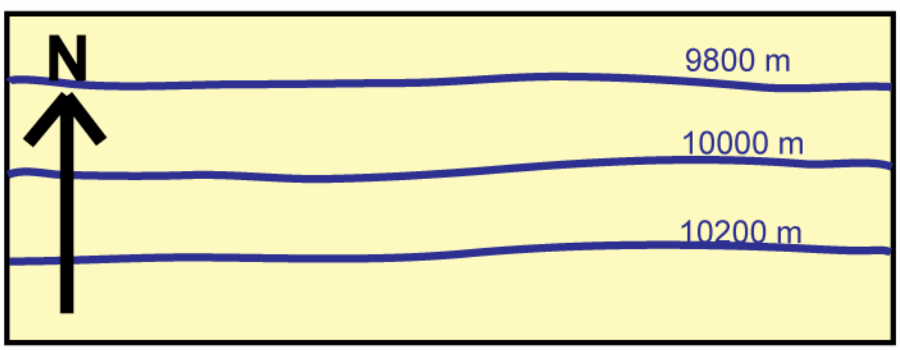
The Coriolis parameter, f, pops up frequently when dealing with large-scale and medium-scale atmospheric motion. It is defined as



Pressure surfaces aren't precisely level, in general. Thus, pressure changes along a constant height surface, and height changes along a constant pressure surface, as shown in this vertical section.



Constant height surface: Z = 10000 m



Constant pressure surface: p = 300 mb

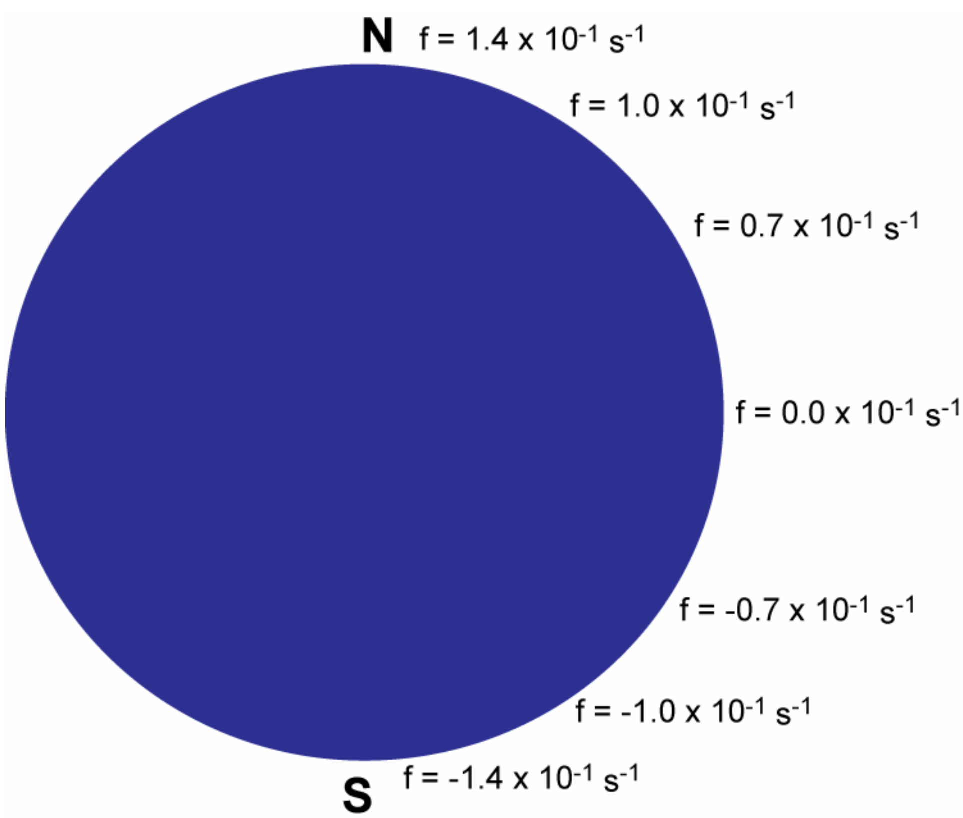
A map of pressure on a height surface shows the pressure decreasing northward; a map of height on a pressure surface shows the height decreasing northward. Notice how the 300 mb isobar and the 10000 m contour are in the same geographical location, corresponding to where the pressure and height surfaces intersect in the vertical section.

$$f \equiv 2\Omega \sin(lat)$$

so it is not a constant but instead depends on latitude. The rotation rate of the Earth, Ω, is not quite constant itself, but it's mighty close. It has the value of 7.292 x 10^{-5} s^{-1} (actually, radians/second, but radians are unitless). This number must seem rather abstract to you, but if you multiply it by the number of seconds in a day, 86,400, you get 2π, saying that the Earth rotates 2π radians (one complete rotation) in one day. Which, after all, is the definition of one day in the first place.

[Those science geeks among you who like to check the math found that you get something just a little bit bigger than 2π. In fact, it's bigger by 366.25/365.25, a correction of less than 1% that's necessary because the Earth is also revolving around the Sun. Once the Earth has completed one entire rotation, it has orbited a bit around the Sun, so to get the Sun in the same position relative to the horizon the Earth has to rotate just a little bit more than one complete rotation.]

Defined as such, f ranges from -1.4 x 10^{-4} s^{-1} at the South Pole to +1.4 x 10^{-4} s^{-1} at the North Pole. It is zero at the equator. At midlatitudes, a commonly-used representative value for f is 1 x 10^{-4} s^{-1}.



The variation of the Coriolis parameter with latitude.

Okay, so now you know *f*, and you already know *g*. Let's plug in some numbers to answer the question: if the height gradient is 3 decameter (30 m) per degree of latitude, how strong would the geostrophic wind be?

$$\left|\vec{v}_g\right| = \frac{g_0}{f}\left|\nabla_h Z\right| = \frac{9.8ms^{-2}1}{1\times10^{-4}s^{-1}} \times \frac{30m}{1^o\,lat} \times \frac{1^o\,lat}{1.11\times10^5 m} = 26.4ms^{-1} = 51kt$$

The last step used the conversion factor of 1.93 from meters per second to knots. A speed of 51 kt is a moderate wind speed, so in the geostrophic sense, a height gradient of 3 dam per degree is a moderate height gradient.

With the equation for the magnitude of the geostrophic wind and our knowledge of the direction of the geostrophic wind, we can write down equations for the individual geostrophic wind components. For example, if we choose our *x* axis to be parallel to the geostrophic wind, the height gradient must be oriented in the negative *y* direction, with lower heights in the direction of positive *y*. So the magnitude of the height gradient is given by the magnitude of the derivative of height with respect to *y*, and to get the wind going in the positive *x* direction we must multiply by -1:

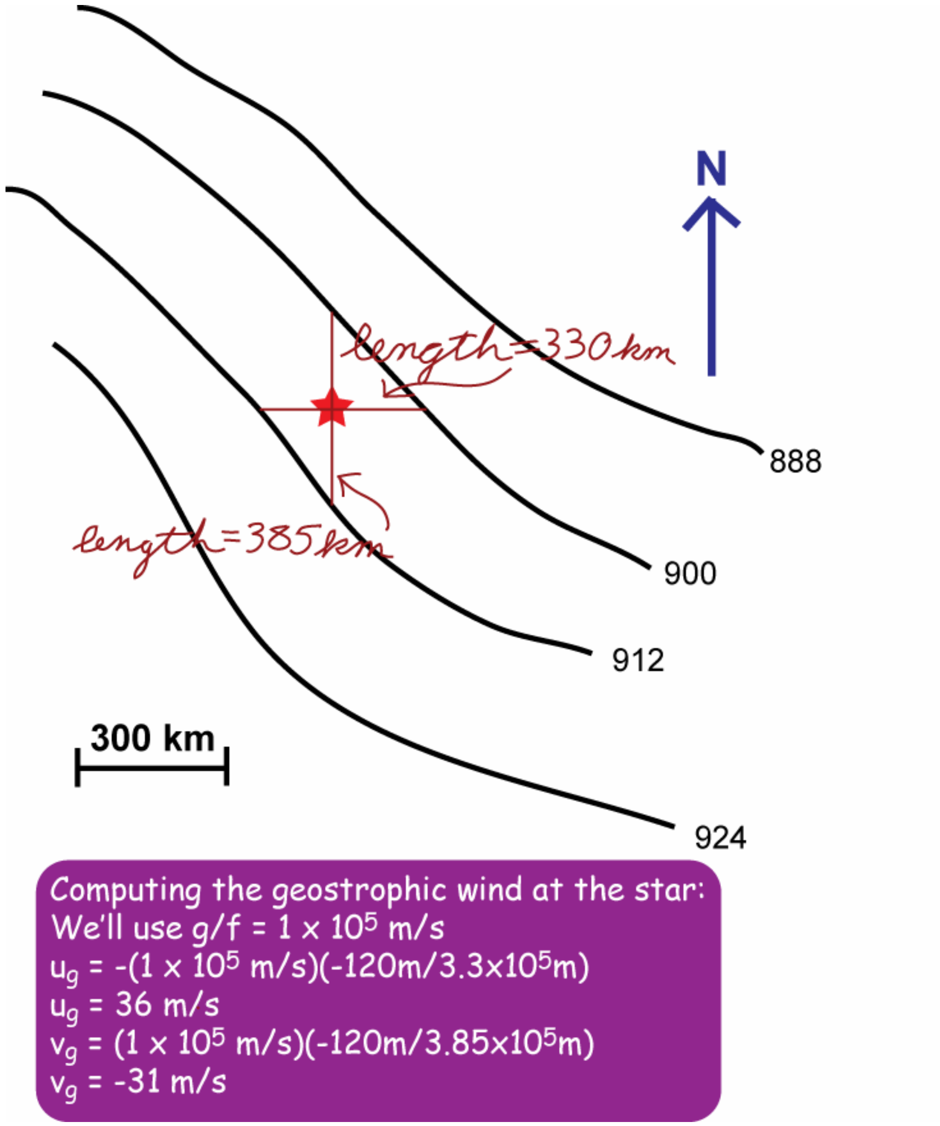
$$u_g = -\frac{g_0}{f}\frac{\partial Z}{\partial y}$$

By the same reasoning, if the geostrophic wind is blowing in the positive *y* direction, heights must increase in the positive *x* direction. So the derivative will already be positive, and no minus sign is needed:

$$v_g = \frac{g_0}{f}\frac{\partial Z}{\partial x}$$

These are the equations defining the two horizontal components of the geostrophic wind. There is no vertical component. They actually are valid in all circumstances, not just when the geostrophic wind is parallel to one of the axes. You can use them to compute the components of the geostrophic wind separately and thereby determine the geostrophic wind vector.

Finally, there is an equation in vector form that defines the geostrophic wind. To determine its form, note that the equation for the *x* component of the geostrophic wind depends on the *y* component of the height gradient, and vice versa. Indeed, the rules describing the geostrophic wind imply that the geostrophic wind is oriented at right angles to the height gradient. So, our vector geostrophic wind equation



must take care of the magnitude (by multiplying the height gradient by *g/f*) and also take care of the direction (by rotating the direction of the height gradient vector by 90°.

In our vector bag of tricks from Chapter 7, there is one vector manipulation that can accomplish the latter task. If we take the cross product of two vectors, the result is a vector that's at right angles to both the original vectors. Now then, if we take a horizontal vector and cross it with a vertical vector, the answer will be a horizontal vector at right angles to the original horizontal vector. And if the vertical vector is the unit vertical vector, the magnitude of the cross product will equal the magnitude of the original vector.

The only thing left is to get the signs right. The answer is:

$$\vec{v}_g = \frac{g_o}{f}\mathbf{k} \times \nabla_h Z$$

Try the Gig 'em rule: hold your arm up over your head, fist pointing skyward, and hand in the shape of a Gig 'em. If the palm of your hand faces higher heights on a pressure surface, your thumb points in the direction of the geostrophic wind. Sure enough, the higher heights would be 90 degrees to the right of the direction the wind is blowing. So the above equation works!

It is also a useful exercise for those who have already begun vector calculus to verify that the two component equations can be derived from the vector equation.

For future reference, here's an equivalent statement of the relationship between geostrophic wind and heights:

$$f\left(\mathbf{k} \times \vec{v}_g\right) = -g_o \nabla_h Z$$

You can verify with your own hand that when you take the cross product of the unit vertical vector and the geostrophic wind, you end up with your thumb pointing toward lower heights 90 degrees to the left of the geostrophic wind, as you should.

So, to summarize, we've got the relationship between geostrophic wind and heights in words and in equation form both for wind components and for the total vector geostrophic wind.